Viscoelasticity of Fluid Dampers

Due to presence of elasticity in damping fluid molecules, such fluids exhibit viscoelastic rather than purely viscous behavior. Flow of the fluid deforms the molecules causing an elastic response, but over time (determined by the viscous drag on the liquid and the stiffness of the molecules) the molecules relax back to their un-deformed state and the elastic response vanishes. With such rheology in damping fluid, realistic viscous dampers are not just viscous but viscoelastic devices modeled best by a series combination of pure viscous dampers and springs, known as Maxwell model shown in Figure 1.

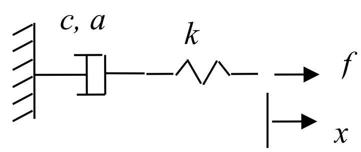

Figure 1 Maxwell model of a fluid viscoelastic damper

The displacement of a realistic damper with viscoelastic trait (fluid viscoelastic damper) is the summation of the displacements in the damper element and spring element, i.e.,

$$x = x_{spring} + x_{damper} \tag{1}$$

Moreover, considering the series arrangement of the damping element and spring element in Maxwell model, the applied force as shown in Equation (2) is the same as the force in each element

$$\begin{aligned} f &= f_{spring} = f_{damper} \\ f &= kx_{spring} = \mathrm{c}\left|\dot{x}_{damper}\right|^{a} sgn\left(\dot{x}_{damper}\right) \end{aligned} \tag{2}$$

Equations (1) and (2) for a fluid viscoelastic damper with linear viscoelastic behavior, i.e. *a*=1, results in a first order linear differential equation

$$t_1\dot{f} + f = c\dot{x} \tag{3}$$

where $t_1 = c/k$, called 'relaxation time', has the unit of time (second) and is the 'time constant' of the first order differential Equation (3).

Fluid viscoelastic dampers

i. with high stiffness behave like a pure viscous damper for which the force is the product of damping coefficient and velocity. This is because the relaxation time approaches zero making the 2nd term in the constitutive model of Equation (5) becoming the dominant term. And
ii. with low stiffness behave for the most part like a spring for which the force is the product of stiffness and displacement. This is because the relaxation time approaches infinity making the first term in Equation (5) the dominant term.

The time traces of spring deflection in response to harmonic stroke as well as the hysteresis plot (contour of force vs. stroke plotted over one cycle) for a nonlinear fluid viscoelastic damper with the velocity exponent of *a*=0.3 and a high spring stiffness of 100 dkN/mm and a low stiffness of 10 kN/mm are presented in Figure 2.

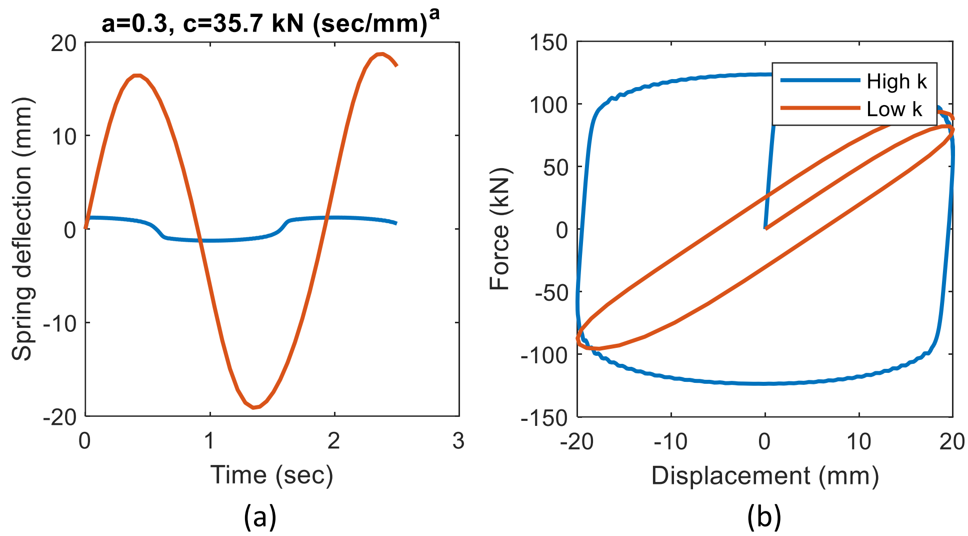

Figure 2 Time traces of spring deflection (a) and hysteresis plot of *f* vs. *x* (b) for a nonlinear fluid viscoelastic damper with two spring stiffness coefficients of 100 kN/mm (High) and 10 kN/mm (Low)

Considering the main purpose of any damper is damping (energy dissipation), fluid viscoelastic dampers are normally designed to have large Maxwell model stiffness.

Dampers are characterized by their response to harmonic strokes, it is of interest to formulate their forced harmonic response at steady state. For linear fluid viscoelastic dampers, this can readily be done by deriving the frequency response function from the first order differential Equation (3), shown by Equation (4)

$$F(\omega) = \frac{j\omega}{t_1(j\omega)+1} X(\omega) \quad (4)$$

where ω is the frequency of harmonic stroke and $F(\omega)$ and $X(\omega)$ are the frequency-domain equivalent of force and stroke, i.e. *f* and *x*. The force *f* at steady state, shown in Equation (5), is evaluated by transforming Equation (4) from frequency-domain to time-domain

$$\begin{aligned} f &= \frac{(\omega t_1)^2}{1+(\omega t_1)^2} k x_0 \sin(\omega t) + \frac{1}{1+(\omega t_1)^2} c \dot{x}_0 \cos(\omega t) \\ f &= \frac{(\omega t_1)^2}{1+(\omega t_1)^2} k x_0 \sin(\omega t) + \frac{(\omega t_1)}{1+(\omega t_1)^2} k x_0 \cos(\omega t) \\ f &= k x_0 \left[\frac{(\omega t_1)^2}{1+(\omega t_1)^2} \sin(\omega t) + \frac{(\omega t_1)}{1+(\omega t_1)^2} \cos(\omega t)\right] = k \frac{(\omega t_1)^2}{1+(\omega t_1)^2} x_0 \sin(\omega t) + c \frac{1}{1+(\omega t_1)^2} v_0 \cos(\omega t) \end{aligned} \quad (5)$$

where x_0 and v_0 are the amplitudes displacement and velocity of the damper.

Equation (5) is similar to the steady-state solution for a system with parallel combination of spring and viscous damper; the model of such system is known as the Kelvin-Voight model shown by Figure 3. As such Kelvin-Voight model with frequency-dependent stiffness and damping ratio is commonly used as an alternative to Maxwell model. Note that in a pure, frequency-independent Kelvin-Voight model the harmonic force/stroke relationship is

$$f = Kx_0 \sin(\omega t) + Cv_0 \cos(\omega t)$$

where $K = k\frac{(\omega t_1)^2}{1+(\omega t_1)^2}$ and $C = c\frac{1}{1+(\omega t_1)^2}$.

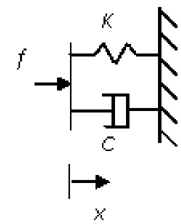

Figure 3 Kelvin-Voight model